TO STUDY SOME PROPERTIES OF FRACTIONAL CALCULUS OPERATORS INVOLVING ANALYTIC FUNCTION AND IT'S APPLICATIONS

A

SYNOPSIS SUBMITTED

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MAHARSHI DAYANAND SARASWATI UNIVERSITY AJMER

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SYNOPSIS

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1. Name of scholar:-

In English **Radhe Shyam Prajapat**

In Hindi राधेश्याम प्रजापत

2. Title of Investigation

In English :-

To Study Some Properties of Fractional Calculus Operators involving analytic function and It's Applications .

In Hindi

भिन्न कलन संकारको के कुछ गुणधर्मो का विश्लेषिक फलन सहित अध्ययन तथा अनुप्रयोग

3. **Location:-**

(a) Department of mathematics M.L.V. Govt. College, Bhilwara

(b) Geographical area of investigations Library and Internet

4. **Introduction, Importance of proposed Investigation, objective**

Introduction:-

Fractional calculus is field of mathematical studies that grows out of the traditional definitions of calculus with differentiation, integration and derivative operators in much the same way fractional exponents with integer value. Fractional calculus is branch of mathematics in which the study of differentiation and integration to an arbitrary order. The concept of exponent is a short notation of repeated multiplication of integer value as par the studies at primary level. However, this physical definition can clearly become confused when considering exponents of non integer value. While almost anyone can verify that $x^3=x.x.x$ how

might one describe the physical meaning of $x^{3.4}$ or moreover the transcendental exponent x^{π} one cannot conceive what it might be like to multiply a number or quantity by itself 3.4 time or π time and yet these expressions have a definite value for any value x verifiable by infinite series expansion.

Thus concept of Fractional calculus is not new. The word Fractional calculus was created when (1695) Hospital asked question as to meaning of $\frac{d^n y}{dx^n}$ if n= $\frac{1}{2}$; that is" what n is Fractional Leibniz's response: "An apparent paradox, form which one day useful consequences will be drown"

Importance of proposed work:-

Fractional calculus has gained considerable popularity and importance during the past few decades, due mainly to its demonstrated applications numerous fields of science and technology. It does indeed provided several potentially useful tool for solving differential equations, integral introdifferential equations, differential subordination and superordination, partial differential equation and various other problems involving analytic functions.

The subject of fractional calculus has applications in diverse and widespread fields of engineering and science such as electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, biological population models, optics, and signals processing. It has been used to model physical and engineering processes that are found to be best described fractional differential equations. The fractional derivative models are used for accurate modelling of those systems that require accurate modelling of damping. In these fields, various analytical and numerical methods including their applications to new problems have been proposed in recent years.

At present, the use of fractional order partial differential equation in real-physical systems is commonly encountered in the fields of science and engineering. The efficient computational tools are required for analytical and numerical approximations of such physical models.

An extremely use of some special functions like Hypergeometric functions played another crucial role in some significant recent developments in the geometric function theory

It is commonly expected that most of the result which will obtain in this work may be useful in the solution of problems like as fractional calculus operator, physical science, engineering and statistics.

Objectives:-

In the proposed study we are going to treat problems both classical as well as applied, related to concerning research area. The broad outlines of objectives of the study as follows:-

1. Study of certain classes of analytic function involving fractional calculus operators and convolution structure.
2. Study of fractional calculus operators associated with probability distributions and its application.
3. Study of fractional calculus operators in integral and differential equation & their applications.
4. Some results associated with Fractional calculus operators involving applied statistics.
5. Integral transform of fractional calculus operator involving special function.
6. Study of unification approach and inequalities for fractional calculus operators of analytic function.

5. Review of the work:-

Fractional calculus developed in 1695 in the result of the brilliant idea of Gottfried Leibniz and it's started from some speculations Euler (1730), Laplace (1812), Lacroix (1819), Fourier(1822), Abel (1823), Keimann Liouwille supported to progress in the Fractional calculus. The theory of univalent functions is a classical problem of complex analysis which belongs to one of the most beautiful subjects in geometric functions theory. It deals with the geometric properties of regular functions, found around the quirk of the 20th century. In spite of the famous coefficient problem the Biberbach conjecture which was solved by (Branges, 1985). In the present epoch, fractional calculus plays an important role in various fields and useful for all the field by its concept of nanotechnology and we can refer it's adoption in viscoelasticity electric transmission lines, ultrasonic wave propagation modeling of speech signals modeling the cardiac tissue electrode interface sound waves propagation in rigid porous materials Diffusion and robotics, genetic algorithms, dynamic modeling, control theory, lateral and longitudinal control of autonomous vehicles, edge detection fluid mechanics, nuclear magnetic resonance, Based on the wide applications in engineering and sciences such as physics, mechanics, chemistry and biology research on Fractional ordinary or partial differential equation and other relative topic is active and extensive around the world. And generalized moment generating function and the fractional moment of probability distribution function by Stephen Wolfe and the fractional moment of a quadratic form in non central normal random distribution is defined in the article of Harvey(1965). In the past few years, the increase of the subject is witnessed by hundred of research papers, several monographs and many international conferences Ross organized the first conference on fractional calculus and its application at New Haven in June 1974 and edited its proceedings. Keith Oldhalm and Jerome

Spainer published the first monograph devoted to fractional calculus in 1974. Some eminent mathematicians and researchers have been solved classical & applied numerous problems by using the operators of fractional calculus in univalent/multivalent analytic functions and obtaining several properties such as coefficient estimates, extreme points, radii of convexity, distortion theorem starlikeness and convexity. Many researcher Raina and Bapna(1999,2003,2004,2009), Shrivastava and Owa(1978,1987,1986,), Saigo(1989), Bapna and jain(2008), Bapna and Mathur(2012) studied on generalization of some historical development and worked on fractional differential Equation and integral inequalities using Riemann-Liouville, Caputo and q-Fractional integrals, progress of fractional calculus operators appears in the books of Kiryakova, Duren(1977,1983), Baeumer and Mark(2005), Silverman, Miller and B.Ross(1974,1975), Nishirnoto(1989,1991,1996), Kilbas(1993), Keith, Oldham and Spainer(1974) old and recent history of fractional and applied analysis can be taken by its poster which is described by Tenreiro Machado, Virginia Kiryakova, Francesco Mainardi at all(2010), In special function researches developed some new and interesting results for hypergeometric function of one and more variables and confluent forms generalization of Mittag-Leffler's function. Apell Hypergeometric function generatized Vsoigt function and K- function etc. These result involve the integral relations, fractional integrals, derivatives, differential equations Laplace and Millin transforms, generalized hypergeometric series form. During last few years a number of research papers have devoted on application of fractional calculus operators. The works of Agnieszka and Torres(2012,2015), Liu(2015), Nair& Pathan(2014), Murugusundaramoorthy and Thilagavathi(2014) , Srivastava(1987,2014,2015), Eker and Rosihan(2015), Bapna and Mathur(2012), Katugampola(2011,2014), Garg and Shekhawat(2015),

Liu(2015), Stanciu(2014), Ramirez Lynnette(2010) are worth mentioning contribution fractional calculus operator and applications.

6. Research gap identified in the proposed field of Investigation:-

The rudiments of the theory had already emerged at beginning of 20th century in the investigation of Koebe in 1907. Different definition of fractional calculus was given by Euler(1730), Fourier(1820) Abel(1823 Lioville(1832) Riemann(1847). Sonin(1869),Lentnikov(1872) H.Laurent(1884), Nekrasove(1888), Caputo(1967) Miller, Ross(1993). The first use of fractional operators was made by, Niels. Abel in 1823. Abel applied the fractional calculus in the solution of integral equations that arises in the formulation of tautocrone problem. A fairly large number of books dedicated to fractional calculus and its application are written by Oldham and Spanier(1974), Bride(1979), Samko, Kilbas and Marichev(1993), Kiryakova(1994), Miller and Ross (1993), Podlubny(1999), Hilfer(2000), Kilbas, Srivastava and Trujillo (2006), and Sabatier, Agrawal and Tenreiro(2007). These books give a vivid account of development of the subject and its applications in diverse fields of science and technology such as fluid flow, diffusion theory, electrical networks and probability scattering theory, transport theory electrochemistry of corrosion visco-elasticity, statistics, and chemical, physics, Fractional calculus is applicable in deriving the solution of certain integral equation involving special functions of mathematical physics. Application of fractional calculus finds in numerical analysis and different area of physics and Engineering in this respect. A Carpinteri and Manardi have edited the present book as .

"Fractals and fractional calculus in continuum mechanics"

Keith. Oldham and Spainer and to investigate inclusion, radius and other various properties of subjects of multivalent function, which are defined by using an extended version of Owa Srivastva fractional differential operator $\Omega(\lambda, \rho)$ and generalizing the applications of differential subordination and superordination analytic function . In 1978 Saiga defined a pair of fractional integral and differential operators involving causes hypergeometric function. Kriyakova in 1986 gave definition of fractional calculus operators with Meijer's G- Function and Fox H-function. Several other researchers namely Kilbas and Saigo (1998), Saigo and Saxena (1998) and Khan, Rehman and Henderson (2011) have also defined and generalized fractional calculus operators. In 1980-2000 a modest amount of published work appeared on the subject of the fractional calculus.

7 Major Hypotheses:-

A hypothesis is refers to a provisional idea whose merit requires evaluation. For proper evaluation, hypothesis should be defined in specific term using operations. A hypothesis is used to formulate provisional data within research work. The formulated hypothesis is the evaluated where either the hypothesis is proven to be true or false through a verifiability or falsifiability. If a hypothesis fails then it need to amend. Testing of scientific hypothesis mainly requires experimentation and in mathematics is being done by deductive reasoning.

In mathematics hypothesis started with unproven statement which is supported by available data and initial ordinary result.

For our proposed research work we start hypothesis using already existing fractional calculus operator's properties and obtain new properties as classes and subclasses that verify existing properties.

8. Research Methodology:-

We shall apply the method integral transform and its inverse to find the distribution sum of n independent random variable and the distribution of product of n independent random variable and can use convolution theorem, distortion theorem etc.

To investigate differential subordination in fractional operators theory.

To evaluate some unification result for fractional calculus operator

To deduce result of probability distribution in fractional calculus operator theory.

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